§ 3.2: Dividing Polynomials

Long Division of Polynomials

Example 1	Long Division of Polynomials

Divide $6x^2 - 26x + 12$ by $x - 4$.

Division Algorithm

If $P(x)$ and D(x) are polynomials with $D(x) \neq 0$, then there exist unique polynomials $Q(x)$ and $R(x)$, where $R(x)$ is either 0 or of degree less than the degree of $D(x)$, such that

$$P(x) = D(x) \cdot Q(x) + R(x).$$

The polynomials $P(x)$ and $D(x)$ are called the **dividend** and **divisor**, respectively, $Q(x)$ is the **quotient**, and $R(x)$ is the **remainder**.

Example 2	Long Division of Polynomials

Let $P(x) = 8x^4 + 6x^2 - 4x + 5$ and $D(x) = 4x^2 - x - 2$. Find polynomials $Q(x)$ and $R(x)$ such that $P(x) = D(x) \cdot Q(x) + R(x)$.

Synthetic Division

Synthetic division is a quick way to divide polynomials when the divisor is of the form $x - c$. In synthetic division we only write down the essential parts of the long division.

Example 3	Synthetic Division

Use synthetic division to divide $2x^2 - 7x^2 + 5$ by $x + 3$.

The Remainder Theorem

Remainder Theorem

If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

Proof:

Example 4	Using the Remainder Theorem to Find the Value of a Polynomial

Let $P(x) = 4x^5 - 2x^4 + 3x - 5$.

(a) Find the quotient and remainder when $P(x)$ is divided by x+5.
(b) Use the remainder theorem to find $P(-5)$.

Factor Theorem

c is a zero of P if and only if $x - c$ is a factor of $P(x)$.

Proof:

Example 5	Factoring a Polynomial Using the Factor Theorem

Let $P(x) = x^3 - 7x + 6$. Show that $P(1) = 0$, and use this fact to factor $P(x)$ completely.

Example 6	Finding a Polynomial With Specified Zeros

Find a polynomial of degree 4 that has zeros -3, 0, and 2 only.

Homework

Due:______________________________

2 – 62 (even)